

**THE VÁZQUEZ MAXIMUM PRINCIPLE
AND THE LANDIS CONJECTURE FOR
ELLIPTIC PDE WITH UNBOUNDED COEFFICIENTS**

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Abstract (joint work with Boyan Sirakov). We develop a new, unified approach to the following two classical questions on elliptic PDE:

- the strong maximum principle for equations with non-Lipschitz nonlinearities,
- the at most exponential decay of solutions in the whole space or exterior domains.

Our results apply to divergence and non-divergence operators with locally unbounded lower-order coefficients, in a number of situations where all previous results required bounded ingredients. Our approach, which allows for relatively simple and short proofs, is based on a (weak) Harnack inequality with optimal dependence of the constants in the lower-order terms of the equation and the size of the domain, which we establish.