

Action versus energy ground states in nonlinear Schrödinger equations

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We consider the stationary nonlinear Schrödinger equation

$$-\Delta u + |u|^{p-2}u = \lambda u \quad \text{in } H_0^1(\Omega),$$

where Ω is a (possibly unbounded) open subset of \mathbf{R}^N and λ is a real parameter. The existence of positive solutions can be addressed by variational methods in at least two different ways: either by minimizing the action functional on the Nehari manifold or by minimizing the energy functional on the set of functions having a prescribed L^2 norm. In the former case, we speak of action ground states, and in the latter of energy ground states. These two approaches are clearly intertwined, since any action ground state is also a critical point of the energy with the appropriate mass constraint and, conversely, any energy ground state is also a critical point of the action, with the appropriate value of λ . Despite these relationships, however, the precise interplay between the action approach and the energy approach (in particular, the question whether an action ground state is necessarily also an energy ground state, or the other way round) has not been thoroughly investigated yet, and the present talk aims at taking some steps in this direction.