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A rather strange weak lower semi-continuity result

In this talk I will present joint work with Antonio Gaudiello (Università di Caserta) and Olivier Guibé (Université de Rouen), concerning a rather strange weak lower semi-continuity result. This result plays an important role in our work on the Neumann's brush, i.e. when passing to the limit in a sequence of Neumann's boundary value problems posed in a sequence of open sets with oscillating boundaries which have the forms of brushes.

The model of this result is the following Lemma:

Let X be a measurable set of R^N, and let y^ε be a sequence of vectors in $(L^2(X))^k$ which satisfy

$$y^\varepsilon = 0 \ \text{ in } X \backslash S^\varepsilon,$$

where S^ε is a sequence of measurable sets whose characteristic functions satisfy, for some function θ,

$$\chi_{S^\varepsilon} \rightharpoonup \theta \ \text{ weakly star in } \ L^\infty(X).$$

Assume that, for some function y,

$$y^\varepsilon \rightharpoonup \theta\, y \ \text{ weakly in } \ (L^2(X))^k.$$

Then

$$\liminf \int_X |y^\varepsilon|^2 \geq \int_X \theta\, |y|^2.$$

Moreover one has the following corrector result:
If one further assumes that

$$\int_X |y^\varepsilon|^2 \to \int_X \theta\, |y|^2,$$

and if E is a measurable subset of X such that y belongs to $(L^2(E))^k$,
then

$$y^\varepsilon = \chi_{S^\varepsilon}\, y + r^\varepsilon, \ \text{ where } \ r^\varepsilon \to 0 \ \text{ strongly in } \ (L^2(E))^k.$$

Our work is a generalization of works of A. Donato & A. Nabil and of A. Corbo Esposito, P. Donato, A. Gaudiello & C. Picard.