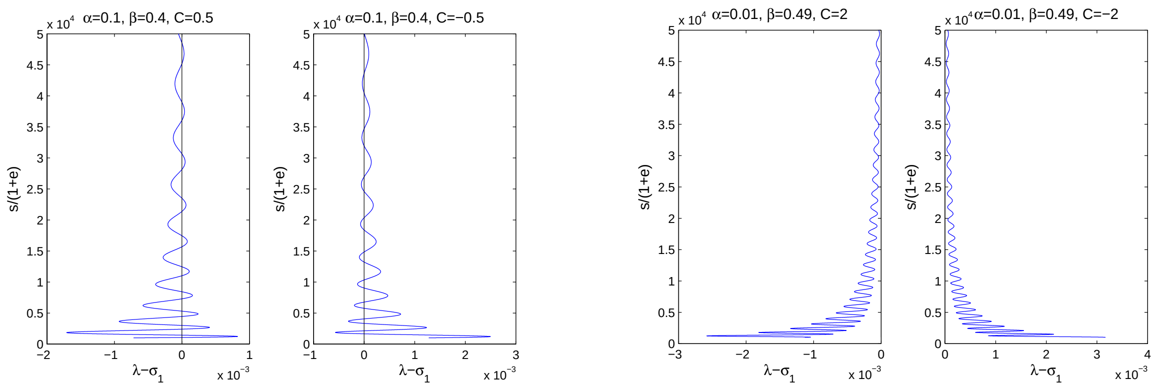
Oscillatory nonlinearities in elliptic equations

Rosa Pardo

I will present some results concerning existence of positive solutions to some elliptic problems with oscillatory boundary conditions

$$-\Delta u + u = 0 \quad \text{in } \Omega, \qquad \frac{\partial u}{\partial \eta} = \lambda u + u^{\alpha}\left[\sin\left(\left|\frac{u}{\Phi_1(x)}\right|^{\beta}\right) + C\right] \quad \text{on} \quad \partial\Omega\,, \qquad (P_{\rm NL})$$

where $\beta > 0$, and $C \in \mathbb{R}$, and (σ_1, Φ_1) stands for the first eigen-pair of the Steklov eigenvalue problem $-\Delta\Phi + \Phi = 0$ in Ω, and $\frac{\partial\Phi}{\partial n} = \sigma\Phi$ on $\partial\Omega$. The case $\alpha < 1$ is asymptotically linear, and corresponds to a *bifurcation from infinity* phenomenon, see [1, 3]. When $|C| < 1$, in [1] we proved the existence of and unbounded sequence of resonant solutions, an unbounded sequence of stable solutions, an unbounded sequence of unstable solutions, and also an unbounded sequence of turning points, see the bifurcation diagrams in Fig. 1 (a)-(b). When $|C| > 1$ in [3] we proved the existence of unbounded sequences of stable solutions, unstable solutions, and turning points, even in the absence of resonant solutions, see Fig. 1 (c)-(d).



$(a) - (b)$ $(c) - (d)$

Figure 1: (a)-(b) Two bifurcation diagrams having infinitely many sub-critical solutions $(\lambda < \sigma_1)$, super-critical solutions $(\lambda > \sigma_1)$, stable solutions, unstable solutions, turning points and resonant solutions $(\lambda = \sigma_1)$ for $|C| < 1$. (a) $C = 0.5$, (b) $C = -0.5$. (c)-(d) Two bifurcation diagrams of stable and unstable solutions, on the left all of them are subcritical, on the right all of them are supercritical, and none is resonant, for $|C| > 1$. (c) $C = 2$ on the left, and (d) $C = -2$.

The case with $\alpha > 1$ corresponds to a *bifurcation from zero* phenomenon, see [2]. In [4] is treated the case of oscillatory nonlinearity at the interior with Neumann boundary conditions. Similar phenomena to the already mentioned can be observed.

References

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